MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

EMT1026 – ENGINEERING MATHEMATICS II

(All Sections / Groups)

29 MAY 2017

9.00 AM – 11.00 AM

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **9 pages** (including cover page) with **4** Questions only.

2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

4. Several tables are provided in the **Appendix** for your reference.

Question 1

(a) Consider a second-order ordinary differential equation:

$$y'' - 5y' + 6y = r(x).$$

(i) If $r(x) = 0$, show that $y = e^{2x}$ and $y = e^{3x}$ are the solutions for the above differential equation.

(ii) If $r(x) = x^2$, find the general solution for the above differential equation. Then, determine the particular solution that satisfies the initial conditions $y(0) = 0$ and $y'(0) = 0$.

[13 marks]

(b) Find the first four nonzero terms in the power series solution about $x_0 = 0$ of the following second-order ordinary differential equation:

$$y'' - xy = 0.$$

Then, determine the unknowns in the solution if $y(0) = 2$ and $y'(0) = 1$.

[12 marks]

Question 2

Consider the following partial differential equation:

$$4u_{xx} - u_t = 0, \quad 0 \le x \le 4, \text{ for } t > 0$$

with boundary conditions $u(0,t) = C_1$ and $u(4,t) = C_2$, where C_1 and C_2 are constants.

(a) If $C_1 = C_2 = 0$, find the general solution for the above partial differential equation by using the separation of variables method.

[18 marks]

(b) If $C_1 = 1$, $C_2 = 0$ and initial condition is $u(x,0) = \sin 3\pi x$, decompose the given problem into a homogeneous PDE and an ODE by letting $u(x,t) = w(x,t) + v(x)$. (**Note**: Do not solve the PDE and ODE found.)

[7 marks]

Continued...

Question 3

(**Note**: The tables in APPENDIX A and APPENDIX B may be useful for solving this question.)

(a) Let $g(t)=e^{3t}$ and $h(t)=t\cos 3t$.

(i) Find the Laplace transform of $h(t)$. [4 marks]

(ii) Hence, show that $L\{g(t)*h(t)\}=\dfrac{s+3}{(s^2+9)^2}$, where $*$ denotes convolution. [4 marks]

(b) An s-domain function is defined by

$$F(s)=\frac{1}{s(s+2)}.$$

(i) Determine the inverse Laplace transform of $F(s)$. [4 marks]

(ii) Rework part (i), but using a *different* transform property this time. Show that your final answer is still the same as that in part (i). [4 marks]

(c) Consider the following ordinary differential equation:

$y''(t)-10y'(t)+25y(t)=0$, subject to $y(0)=\frac{1}{5}$ and $y'(0)=2$.

By applying Laplace transform, show that

$$L\{y(t)\}=Y(s)=\frac{s}{5(s-5)^2}.$$

Hence, obtain the complete solution of the above differential equation. [9 marks]

Continued...

Question 4

(**Note**: The tables in APPENDIX C, APPENDIX D and APPENDIX E may be useful for solving this question.)

(a) In a very large shipment of integrated-circuit chips, the probability that any one chip is defective is 0.10. If 20 chips are randomly sampled from the shipment,

(i) find the probability that the sample has no defective chip. [4 marks]

(ii) find the probability that at least 3 chips in the sample are defective. [3 marks]

(b) At a ketchup production factory, dispensers are calibrated to release an average of 250ml of ketchup per bottle. With the current model of dispensers, the amount of ketchup each bottle receives is normally distributed with a standard deviation of 7ml. Bottles with less than 235ml of ketchup are considered underfilled. And those with more than 265ml are considered overfilled.

(i) What proportion of bottles have an acceptable volume of ketchup? [5 marks]

(ii) The company is considering to replace its dispensers with a different model that releases ketchup according to a uniform distribution, $U(234,266)$. Determine whether this will increase the proportion of acceptable ketchup bottles. [4 marks]

(c) Let X be a continuous random variable with probability density function (pdf)

$$f(x)=\begin{cases} k(4-x), & 0 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that $k=1/8$. [3 marks]

(ii) Determine the variance of X. [6 marks]

Continued...

APPENDIX A:

Common Laplace Transform Pairs

	$f(t)$	$F(s) = L\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	t^n	$\frac{n!}{s^{n+1}}$, $n = 1,2,\ldots$
4.	e^{at}	$\frac{1}{s-a}$
5.	$\cos at$	$\frac{s}{s^2+a^2}$
6.	$\sin at$	$\frac{a}{s^2+a^2}$
7.	$\cosh at$	$\frac{s}{s^2-a^2}$
8.	$\sinh at$	$\frac{a}{s^2-a^2}$
9.	$u(t-a)$	$\frac{e^{-as}}{s}$
10.	$\delta(t-a)$	e^{-as}

Continued...

APPENDIX B:

Laplace Transform Properties

	Property Name	Formula
1.	Linearity	$L\{af(t)+bg(t)\}=aL\{f(t)\}+bL\{g(t)\}$
2.	s - shifting	$L\{e^{at}f(t)\}=F(s-a)$
3.	Transform of Derivative	$L\{f'(t)\}=sL\{f(t)\}-f(0)$ $L\{f''(t)\}=s^2L\{f(t)\}-sf(0)-f'(0)$
4.	Transform of Integration	$L\left\{\int_0^t f(\tau)d\tau\right\}=\frac{1}{s}F(s)$
5.	t - shifting	$L\{u(t-a)f(t-a)\}=e^{-as}F(s)$
6.	Differentiation of Transform	$L\{t.f(t)\}=-F'(s)$
7.	Integration of Transform	$L\left\{\frac{f(t)}{t}\right\}=\int_s^\infty F(\tilde{s})d\tilde{s}$
8.	Convolution Theorem	$L\{f(t)*g(t)\}=F(s).G(s)$, where $f(t)*g(t)=\int_0^t f(\tau)g(t-\tau)d\tau$.

Continued…

APPENDIX C:

Special Discrete Probability Distributions

Binomial Distribution, $X \sim b(x; n, p)$	
P.m.f.	$P(X = x) = {}^{n}C_{x} p^{x} q^{n-x}$, for $x = 0,1,\ldots,n$ and where $q = 1 - p$.
Mean	$E[X] = np$
Variance	$Var(X) = npq$
Hypergeometric Distribution, $X \sim h(x; N, n, k)$	
P.m.f.	$P(X = x) = \dfrac{{}^{k}C_{x} \times {}^{N-k}C_{n-x}}{{}^{N}C_{n}}$, for $x = 0,1,\ldots \min(n,k)$.
Mean	$E[X] = \dfrac{nk}{N}$
Variance	$Var(X) = \dfrac{nk}{N}\left(\dfrac{N-n}{N-1}\right)\left(1 - \dfrac{k}{N}\right)$
Poisson Distribution, $X \sim p(x; \lambda)$	
P.m.f.	$P(X = x) = \dfrac{\lambda^{x} e^{-\lambda}}{x!}$, for $x = 0,1,\ldots$.
Mean	$E[X] = \lambda$
Variance	$Var(X) = \lambda$

Continued...

APPENDIX D:

Special Continuous Probability Distributions

Continuous Uniform Distribution, $X \sim U(a,b)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \dfrac{a+b}{2}$
Variance	$Var(X) = \dfrac{(b-a)^2}{12}$
Exponential Distribution, $X \sim \exp(1/\beta)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{\beta} e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \beta$
Variance	$Var(X) = \beta^2$
Normal Distribution, $X \sim N(\mu,\sigma)$	
If X is any normal random variable where $X \sim N(\mu,\sigma)$, then the transformation $$Z = \frac{X-\mu}{\sigma}$$ yields a *standard* normal variable where $Z \sim N(0,1)$.	

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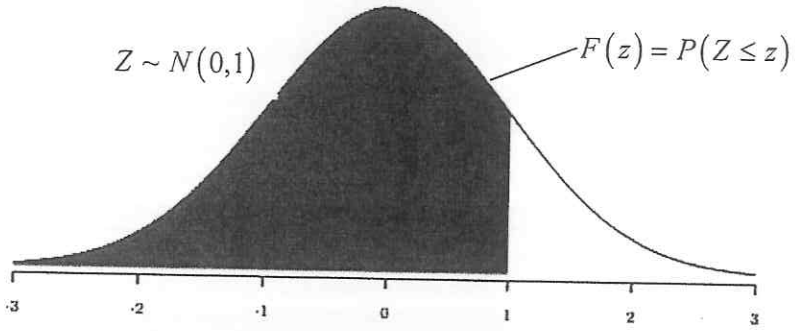
APPENDIX E:

Cumulative Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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